return is the only unknown. Solving for the rate of return we have an expression in terms of property compensation, depreciation, revaluation, property taxes, and asset value, where each term is a sum for residential structures and land:

$$r_t = (\text{Property compensation -- property taxes} - \text{depreciation} + \text{revaluation})/\text{value of capital stock at the end of last period.}$$

We assume that this rate of return is also applicable to owner-utilized consumer durables.

Given the rate of return for household sector assets, we can compute capital service prices for residential structures, land, and consumer durables. We construct a quantity index of household capital input as a Divisia index of the capital services for these three assets. Finally, we compute the implicit price for household sector capital input.

The derivation of capital service prices for assets held by the household sector must be modified for the business enterprise sector due to direct taxation of business property compensation. The general form for capital service price becomes

$$q_{K,t} = \left[\frac{1 - u_t z_t}{1 - u_t}\right] \left[q_{A,t-1} r_t + q_{A,t}\delta - \left(q_{A,t} - q_{A,t-1}\right)\right] + q_{A,t}\tau_t,$$

where u_t is the effective rate of direct taxation on business net income and z_t is the present value of depreciation allowances on a unit of new investment.[8] Depreciation allowances are different from zero only for durables and structures.

We assume that the rate of return is the same for all business assets. Thus we can equate total property compensation to the sum of each capital service price times the lagged capital stock of the corresponding asset. Substituting the capital service price formulas into this expression yields an equation where the rate of return is the only unknown. Solving for the rate of return yields the following expression:

r_t = (Property compensation -- property taxes -- direct taxes -- depreciation + revaluation)/ value of capital stock at the end of last period, where each item is a sum for all six types of business enterprise assets.

Our estimate of the effective rate of business enterprise direct taxes is obtained as the ratio of federal and provincial corporate income taxes less corporate taxes paid by government enterprises to business property income less taxes on business property and the imputed value of depreciation allowances for tax purposes.[9] Imputed depreciation differs from depreciation for tax purposes in reflecting changes in the present value of future depreciation allowances as well as the current flow of depreciation allowances.

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[8] See Hall and Jorgenson (1967), (1971) for derivation of these results.

[9] See Table 1a above for details on tax treatment.

The present value of depreciation deductions on new investment depends on depreciation formulas allowed for tax purposes, the lifetimes of assets used in calculating depreciation, and the rate of return. We assume that the rate of return used for discounting future depreciation allowances in the corporate sector is constant at ten percent. The declining balance depreciation method is permitted by the Canadian tax authorities. Rates are specified for a variety of asset types and industries. We have averaged the specified rates and arrived at the following estimated rates applicable to our aggregates: .05 for nonresidential structures; .30 for machinery and equipment; and .10 for residential structures.

We estimate the price of capital services for each asset employed in the business sector by substituting the business rate of return into the corresponding formula for the price of capital services. These formulas also depend on acquisition prices of capital assets, rates of replacement, and variables describing the tax structure. The quantity index of business capital input is computed as a Divisia index of the quantity of capital services for the six types of assets, where the weights are the relative shares of capital input in total business sector property compensation. Finally, we compute the implicit price for business sector capital input.

We construct the quantity index of capital input for the entire private domestic economy as a Divisia index of the quantity indexes of (1) household and (2) business enterprise capital input. The price index is computed as the ratio of total property compensation divided by the quantity index. In Table 9 we present the price and quantity indexes for capital input in the domestic business economy.

TABLE 9

GROSS PRIVATE DOMESTIC CAPITAL INPUT, 1947-1973 (CONSTANT DOLLARS of 1961)

Year	1. Private Domestic Capital Stock	2. Capital Input Per Unit of Capital Stock	3. Private Domestic Capital Input Price Index	4. Private Domestic Capital Input Quantity Index
1947	65752.3	.796	.079	52331.8
1948	69859.5	.821	.092	57381.9
1949	73379.9	.840	.099	61661.0
1950	77252.8	.857	.107	66178.4
1951	82261.7	.874	.111	71934.8
1952	86930.9	.890	.125	77390.3
1953	91287.5	.906	.123	82673.8
1954	96596.2	.921	.111	88992.3
1955	100314.3	.932	.128	93499.4
1956	105511.8	.945	.131	99723.9
1957	112456.1	.961	.120	108064.0
1958	118304.7	.975	.123	115339.7
1959	123003.5	.985	.125	121110.9
1960	128076.1	.993	.125	127117.8
1961	132658.5	1.000	.123	132658.5
1962	136494.0	1.008	.125	137532.8
1963	141103.1	1.015	.134	143227.4
1964	146130.2	1.025	.144	149804.4
1965	152273.8	1.038	.149	158075.8
1966	159886.1	1.055	.153	168640.5
1967	168120.8	1.074	.143	180549.5
1968	174791.3	1.090	.146	190474.6
1969	181569.7	1.101	.151	199948.6
1970	189285.8	1.112	.153	210446.5
1971	195133.0	1.119	.165	218356.6
1972	202059.4	1.130	.170	228295.2
1973	209911.5	1.146	.196	240535.2

We construct the quantity index of total domestic business sector factor input as a Divisia index of the quantity indexes of (1) labor input and (2) capital input. The price index is computed as the ratio of total factor compensation divided by the quantity index. In Table 10 we present the price and quantity indexes of total factor input, as well as the relative share of property outlay in total factor outlay.

5. Manhour Productivity and Total Factor Productivity

The most commonly employed measure of productivity is the ratio of real output to total manhours of labor input. This measure has the virtue of simplicity but the defect that it may be very poorly related to our view of increases in productivity as increases in the efficiency of the production process. A more satisfactory measure of economic efficiency is total factor productivity, the ratio of real output to a quantity index of the input of all productive factors. In Table 11 we present estimates of manhour and total factor productivity for the Canadian economy. Manhour productivity is th ratio of our quantity index of domestic business production to total manhours. For ease of comparison we normalize this ratio to 1.0 in 1970. Total factor productivity is the ratio of our quantity indexes of domestic business production and domestic business factor input derived in Sections 3 and 4, respectively.

For purposes of comparison we also compute two alternative estimates of total factor productivity. The first variant of total factor productivity

TABLE 10

GROSS DOMESTIC FACTOR INPUT, CANADA, 1947-1973 (CONSTANT DOLLARS of 1961)

Year	1. Gross Private Domestic Factor Input Price Index	2. Gross Private Domestic Factor Input Quantity Index	3. Property Compensation Relative Share
1947	,519	22918,1	.346
1948	.586	23891,7	.376
1949	,618	24817,4	.397
1950	.681	25243,2	,413
1951	,748	26434,3	.406
1952	,819	27428,7	.430
1953	,837	28439,3	.426
1954	,810	29255,2	.418
1955	,883	30175,8	.449
1956	.930	31799,4	.443
1957	,927	33181,8	,423
1958	,948	33838,9	.441
1959	,970	35175,4	.444
1960	,992	36130,9	.444
1961	1,000	36397,6	,447
1962	1,024	37581,9	.448
1963	1,079	38642,9	.460
1964	1,144	40124,6	.470
1965	1,205	41857,6	,467
1966	1.279	43758,7	,462
1967	1,290	45587,8	.440
1968	1.350	46721,4	.442
1969	1.436	48251,5	,435
1970	1,502	49306,6	,434
1971	1,615	50684,4	,440
1972	1,706	52487,9	.434
1973	1.875	55313.5	.456

TABLE 11

MANHOUR AND TOTAL FACTOR PRODUCTIVITY, CANADA, 1947-1973 (1961 = 1.000)

Year	Manhour Productivity	Total Factor Productivity
1947	.527	.797
1948	.541	.792
1949	.564	.805
1950	.637	.872
1951	.652	.866
1952	.710	.913
1953	.747	.934
1954	.751	.902
1955	.811	.954
1956	.860	.996
1957	.876	.981
1958	.918	.986
1959	.934	.990
1960	.958	.992
1961	1.000	1.000
1962	1.037	1.028
1963	1.082	1.056
1964	1.129	1.090
1965	1.180	1.123
1966	1.233	1.147
1967	1.252	1.134
1968	1.324	1.167
1969	1.359	1.177
1970	1.426	1.201
1971	1.475	1.227
1972	1.503	1.235
1973	1.539	1.259

is based on the work of Denison (1962), (1967), which does not take into account the impact of changes in the composition of the aggregate capital stock on factor input. Thus we compute an alternative quantity index of total factor input as a Divisia index of labor input and the aggregate capital stock. The second variant of total factor productivity is based on the work of Solow (1960), which does not take into account changes in the composition of the aggregate capital stock or the labor force. Thus we compute an alternative quantity index of total factor input as a Divisia index of manhours (unadjusted for educational attainment) and capital stock. The resulting two variants of total factor productivity are presented in Table 12. It is clear that failure to account for compositional changes of labor or capital input have a substantial impact on estimates of total factor productivity.

Returning to our preferred measurement of total factor productivity, we note that we can represent the input of capital and labor services as products of terms representing the quality of capital and labor and the quantity of capital and labor:

$$K_S = q_K K_A, \qquad L_S = q_L L_A \quad ,$$

when K_S is the input of capital services, K_A is aggregate capital stock, L_S is the input of labor services, and L_A is the "stock" of manhours used in production. The ratios K_S/K_A and L_S/L_A indicate the quality of K_A and L_A in the sense of services provided per unit of stock. These ratios will change as a result of compositional changes in the stock. They are presented in Table 13, normalized to 1.0 in 1970 for comparison. The labor quality

TABLE 12

TOTAL FACTOR PRODUCTIVITY, CANADA, 1947-1973
(1961 = 1.000)

Year	Labor Services and Capital Stock	Man Hours and Capital Stock
1947	.725	.693
1948	.729	.698
1949	.747	.718
1950	.816	.786
1951	.817	.789
1952	.868	.841
1953	.894	.870
1954	.871	.850
1955	.925	.907
1956	.972	.956
1957	.964	.951
1958	.975	.965
1959	.983	.976
1960	.989	.985
1961	1.000	1.000
1962	1.032	1.035
1963	1.064	1.069
1964	1.103	1.112
1965	1.142	1.154
1966	1.176	1.191
1967	1.171	1.190
1968	1.213	1.236
1969	1.229	1.255
1970	1.260	1.290
1971	1.291	1.326
1972	1.305	1.344
1973	1.339	1.382

TABLE 13

QUALITY OF FACTOR INPUTS, 1947-1973
(1961=1.000)

Year	Labour	Capital
1947	.923	.796
1948	.928	.821
1949	.932	.840
1950	.936	.857
1951	.941	.874
1952	.947	.890
1953	.952	.906
1954	.958	.921
1955	.964	.932
1956	.970	.945
1957	.976	.961
1958	.982	.975
1959	.988	.985
1960	.994	.993
1961	1.000	1.000
1962	1.005	1.008
1963	1.010	1.015
1964	1.015	1.025
1965	1.019	1.038
1966	1.024	1.055
1967	1.029	1.074
1968	1.034	1.090
1969	1.039	1.101
1970	1.044	1.112
1971	1.049	1.119
1972	1.054	1.130
1973	1.059	1.146

index of L is of course the index of educational attainment described in Section 4.

Our measure of total factor productivity assumes that production in the domestic business economy can be closely approximated by the relation

$$Y^* = A^* + \bar{W}_K K_S^* + \bar{W}_L L_S^* ,$$

where Y^* is the rate of growth of gross domestic business product, A^* is the rate of growth of total factor productivity, K_S^* is the rate of growth of capital input, L_S^* is the rate of growth of labor input, $\bar{W}_K$ is the average (over two years) share of property compensation, and $\bar{W}_L$ is the average share of labor compensation. Substituting $K_S = q_K K_A$ and $L_S = q_L L_A$ into this equation yield,

$$Y^* = A^* + \bar{W}_K q_K^* + \bar{W}_K K_A^* + \bar{W}_L q_L^* + \bar{W}_L L_A^*.$$

Now let us denote manhour productivity $M = Y/L_A$. We can write the rate of growth of manhour productivity as $M^* = Y^* - L_A^*$. Finally, substituting in the above expression for Y^* we have

$$M^* = A^* + \bar{W}_L q_L^* + \bar{W}_K q_K^* + \bar{W}_K (K_A^* - L_A^*).$$

Thus we find that total factor productivity can be considered as simply one component in manhour productivity.

Averaged over the time-period 1947-1973 Y^* is .051 while A^* is .018 Thus our estimates imply that 65.8% of the growth in Canadian gross domestic business product is attributable to increases in total factor input, while 34.2% is attributable to increases in total factor productivity. The proportions of the increase in total factor input are presented in Table 14.

TABLE 14

SOURCES OF GROWTH IN REAL FACTOR INPUT: QUANTITY OF LABOUR INPUT ($\bar{w}_L L^*$), QUALITY OF LABOUR INPUT ($\bar{w}_L q_L^*$), QUANTITY OF CAPITAL INPUT ($\bar{w}_K K^*$), AND QUALITY OF CAPITAL INPUT ($\bar{w}_K q_K^*$) AS PROPORTIONS OF THE RATE OF GROWTH OF REAL FACTOR INPUT

Year	$\bar{w}_L L^*$	$\bar{w}_L q_L^*$	$\bar{w}_K K^*$	$\bar{w}_K q_K^*$
1947–1973	.044	.107	.654	.195

TABLE 15

SOURCES OF GROWTH IN MANHOUR PRODUCTIVITY (M^*): TOTAL FACTOR-PRODUCTIVITY (A^*), QUALITY OF LABOUR INPUT ($\bar{w}_L q_L^*$), QUALITY OF CAPITAL INPUT ($\bar{w}_K q_K^*$) AND CAPITAL DEEPENING $\bar{w}_K(K_A^* - L_A^*)$

Year	M^*	A^*	$\bar{w}_L q_L^*$	$\bar{w}_K q_K^*$	$\bar{w}_K(K_A^* - L_A^*)$
1947-1973	.041	.018	.003	.006	.015

Finally, in Table 15 we present the average rate of growth of manhour productivity and its components. Manhour productivity has increased at an average rate of growth of 4.1% per year. Rising total factor productivity accounts for 1.8% of the total, while increases in labor quality account for 0.3%, increases in capital quality account for 0.6% and capital deepening accounts for 1.5%. We conclude that increases in total factor productivity are the most important component of observed increases in manhour productivity, but that capital deepening has also been an important factor.

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